Optimized Occulters
A Comparative Sensitivity Analysis

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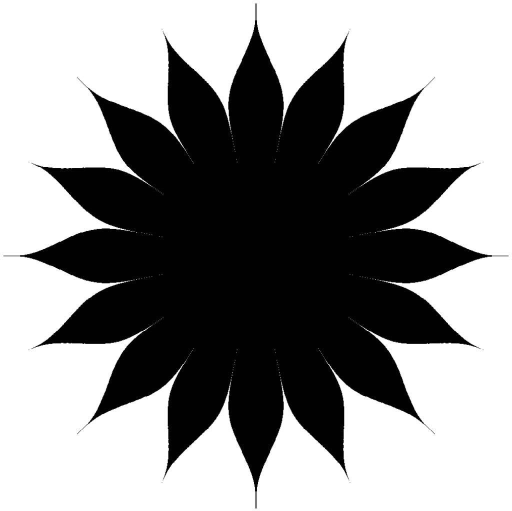
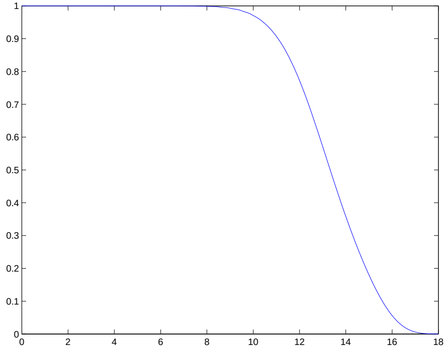
http://www.princeton.edu/∼rvdb

Petal-Shaped Occulters



16-Petal Occulter $A(r,\theta)$



Radial Attenuation $A(r)$

- Babinet's principle plus Fresnel propagation:

$$E(\rho,\phi) = 1 - \frac{1}{i\lambda z}\int_0^\infty \int_0^{2\pi} e^{\frac{i\pi}{\lambda z}(r^2+\rho^2-2r\rho\cos(\theta-\phi))} A(r,\theta) r\, d\theta\, dr.$$

- From Jacobi-Anger expansion we get:

$$\begin{aligned} E(\rho,\phi) &= 1 - \frac{2\pi}{i\lambda z}\int_0^R e^{\frac{i\pi}{\lambda z}(r^2+\rho^2)} J_0\left(\frac{2\pi r\rho}{\lambda z}\right) A(r) r\, dr \\ &\quad - \sum_{k=1}^\infty \frac{2\pi(-1)^k}{i\lambda z}\left(\int_0^R e^{\frac{i\pi}{\lambda z}(r^2+\rho^2)} J_{kN}\left(\frac{2\pi r\rho}{\lambda z}\right)\frac{\sin(\pi k A(r))}{\pi k} r\, dr\right) \\ &\quad \times \left(2\cos(kN(\phi-\frac{\pi}{2}))\right) \end{aligned}$$

where N is the number of petals.

- For small ρ, truncated summation well-approximates full sum.
- Truncated after 10 terms.
- $\lambda \in [0.4, 1.1]$ microns,
- $z = 72,000$ km, $R = 25$ m.

Hypergaussian Profile

$$A(r) = \begin{cases} 1 & \text{if } r \leq a \\ e^{-\left(\frac{r-a}{b}\right)^n} & \text{if } a < r \leq R \\ 0 & \text{if } R < r. \end{cases}$$

Specific choice:

$$a = 9.5, \quad b = 9.5, \quad R = 25, \quad n = 6.$$

BOSS Polynomial Profile

$$A(r) = \begin{cases} 1 & \text{if } r \leq a \\ 1 - \sum_n C_n y^n & \text{if } a < r \leq R \\ 0 & \text{if } R < r. \end{cases}$$

where $y = \frac{(r/R)^2 - \epsilon^2}{1-\epsilon^2}$.

Specific choice:

$$\epsilon = 0.15, \quad C_4 = 35, \quad , C_5 = -84, \quad , C_6 = 70, \quad , C_7 = -20, \quad z = 100,000 \text{ km}.$$

Profile Optimization

$$
\begin{array}{llll}
\text{minimize} & \int_0^R A(r) r dr & & \\
\text{subject to} & -10^{-c} & \leq \Re(E(\rho)) \leq 10^{-c} & \text{for } \rho \in \mathcal{S}, \lambda \in \mathcal{L} \\
& -10^{-c} & \leq \Im(E(\rho)) \leq 10^{-c} & \text{for } \rho \in \mathcal{S}, \lambda \in \mathcal{L} \\
& & A(r) = 1 & \text{for } r \leq a \\
& & A'(r) \leq 0 & \text{for all } r \\
& -d & \leq A''(r) \leq d & \text{for all } r
\end{array}
$$

Specific choice:

$$a = 6.25, \quad R = 25, \quad c = 5.25, \quad d = 0.08, \quad \mathcal{S} = [0, 3], \quad \mathcal{L} = [0.4, 1.1] \times 10^{-6}$$

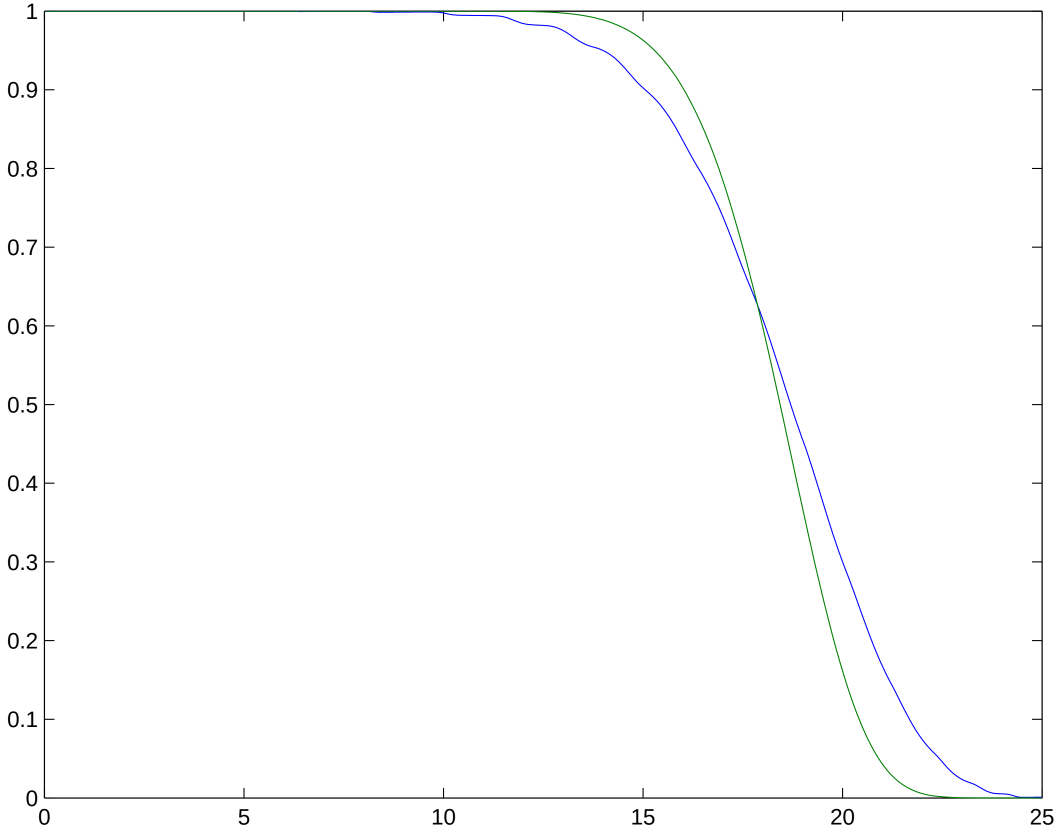
where all quantities are in meters.

An infinite dimensional linear programming problem.
Discretize:

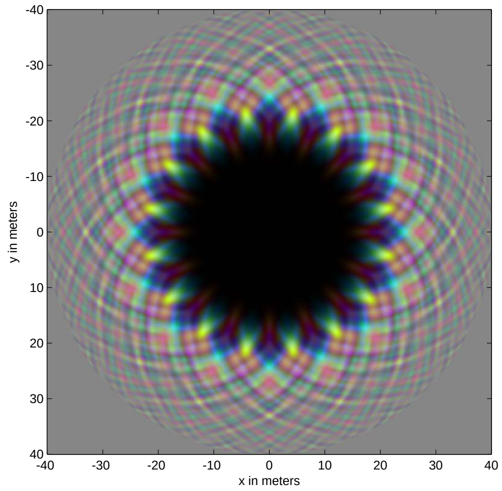
- $[0, R]$ into 3000 evenly space points.
- $\mathcal{S}$ into 18 evenly spaced points.
- $\mathcal{L}$ into increments of 100 nm.

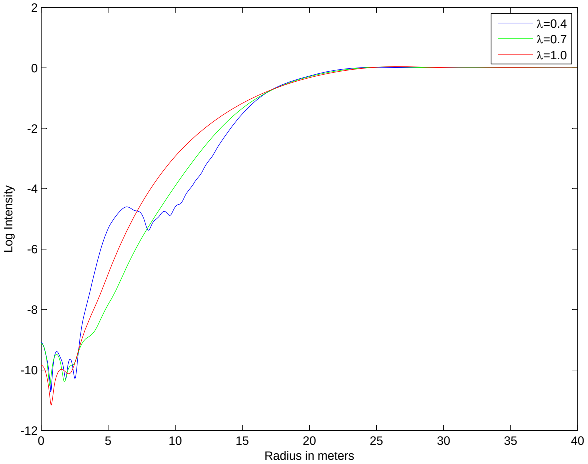
Attenuation Profiles

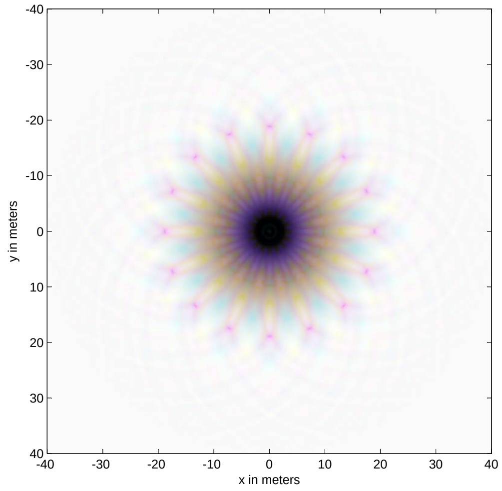
Optimized vs. Hypergaussian

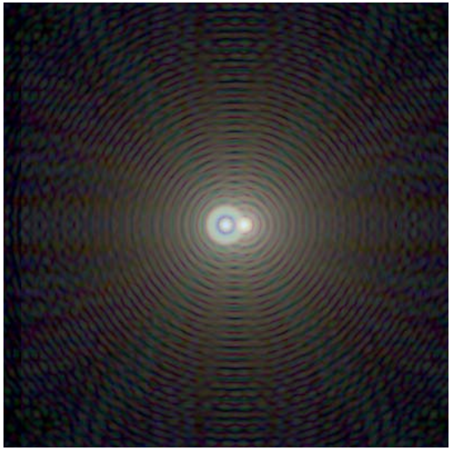


Optimized 16-Petal Occulter

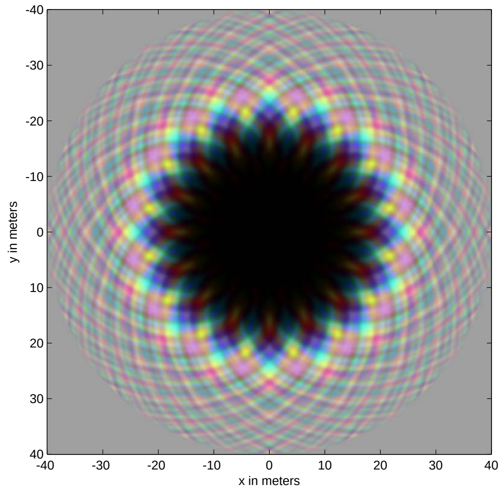


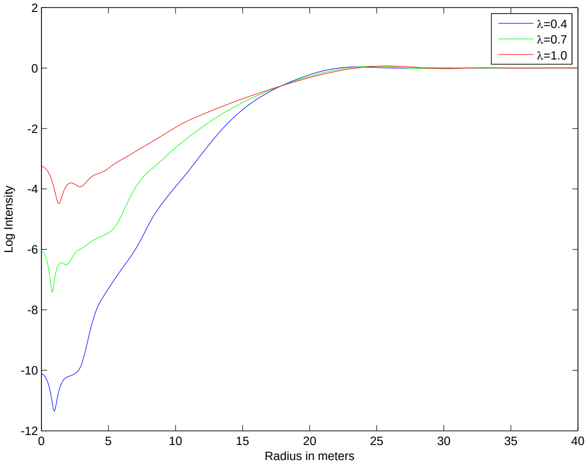


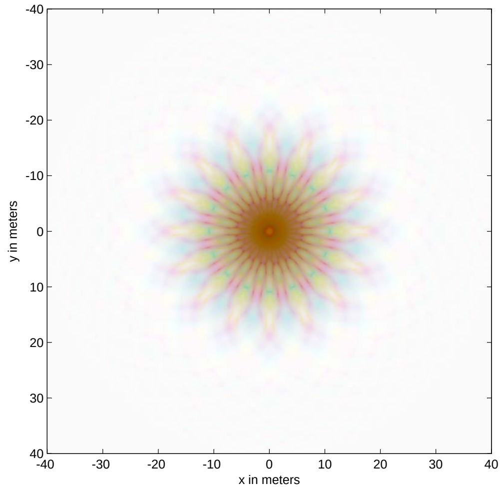


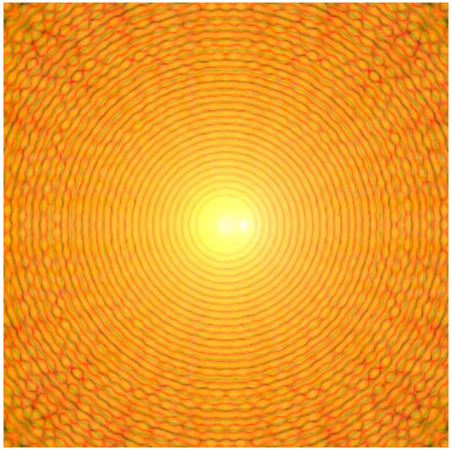


Hypergaussian 16-Petal Occulter

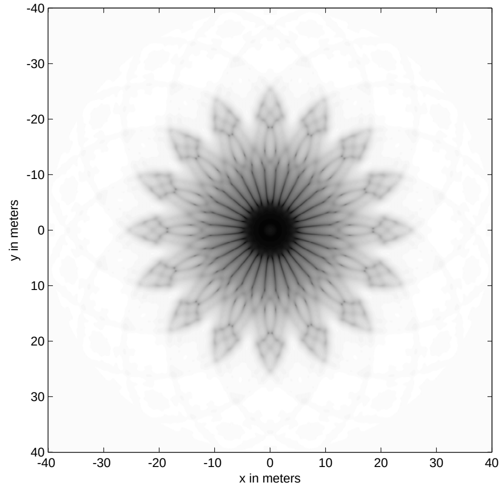


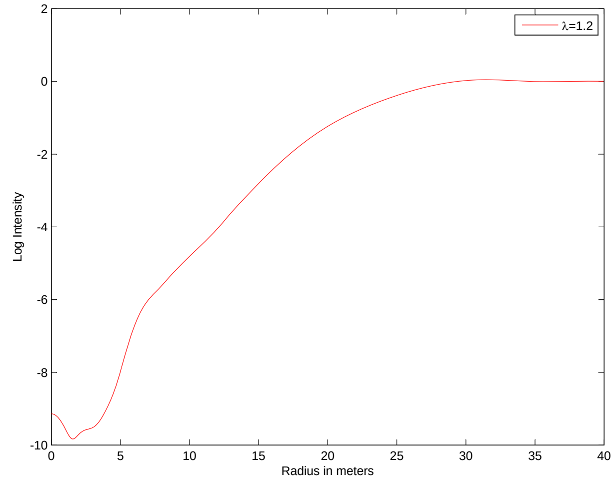


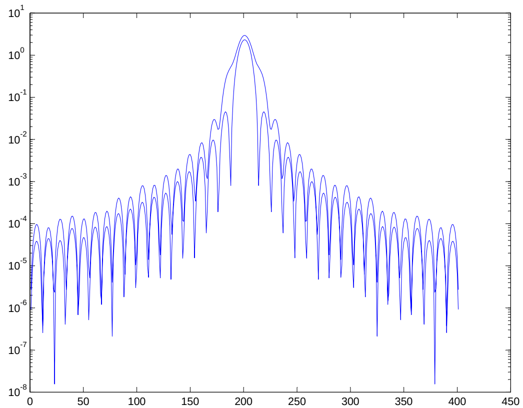


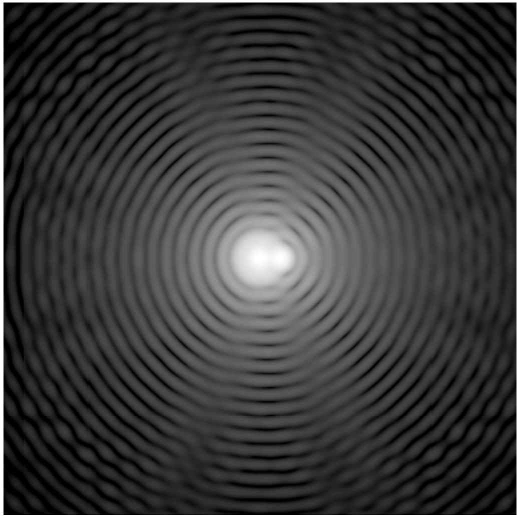


70 m Hypergaussian at 50,000 km









iwa = 150 marcsec

Some Thoughts on Hybrid Designs

To reach 60 marcsec on all target stars, need at least a 25 meter radius shade no closer than 72,000 km. Can a hybrid design relax the requirements (fewer petals, smaller shade, closer shade)?

Consider a pupil mask at the conjugate of the shade, where the starshade is imaged.

- A solid, shaped mask can be used overlapped with the image to create a deeper shadow. This allows relaxation of the shadow requirements of the external shade while still removing most of the light. Also potentially provides a signal for pointing.
- A shaped pupil could be used to create a high contrast PSF.
- A Lyot coronagraph could be used.

Some Questions/Issues to Consider:

- What is the relaxation in the requirements on the starshade?
- What are the alignment and pointing requirements?
- Can it be made to work achromatically? How many channels are required?
- How much will stray light degrade performance?
- Do these issues negate the telescope advantages of the occulter?

Occulter Flight Dynamics